On Consistent Equations for Massive Spin-2 Field Coupled to Gravity in String Theory

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Abstract

We investigate the problem of derivation of consistent equations of motion for the massive spin 2 field interacting with gravity within both field theory and string theory. In field theory we derive the most general classical action with non-minimal couplings in arbitrary spacetime dimension and find the most general gravitational background on which this action describes a consistent theory. We show also that massive spin 2 field allows in principle consistent description in arbitrary background if one builds its action in the form of an infinite series in the inverse mass square. Using sigma-model description of string theory in background fields we obtain in the lowest order in α' the explicit form of effective equations of motion for the massive spin 2 field interacting with gravity from the requirement of quantum Weyl invariance and demonstrate that they coincide with the general form of consistent equations derived in field theory.

String theory contains an infininite number of massive fields with various spins interacting with each other and with a finite number of massless fields and should provide a consistent description of higher spins interaction. Within ordinary field theory consistent classical actions for the higher spin fields are known only on specific curved spacetime manifolds. For example, massive integer spins described by symmetric tensors of corresponding ranks were investigated only in spacetimes of constant curvature [1]–[5][2]. It means that gravity field in these descriptions should not be dynamical since it does not feel the presence of higher spins matter through an energy-momentum tensor. Hence a consistent classical action for gravity and a higher massive field is not known and there are some indications that it does not exist at all.

One of these indications comes from considering a Kaluza-Klein decomposition of Einstein gravity in $D-$dimensional spacetime into gravity plus infinite tower of massive second rank tensor fields in $(D-1)-$dimensional world, masses being proportional to inverse compactification radius. The resulting four dimensional theory of spin 2 fields interacting with gravity and with each other should be consistent as we started from the ordinary Einstein theory and just consider it on a specific manifold. But as was shown in [7], it is impossible to reduce this theory consistently to a finite number of spin 2 fields, i.e. consistency can be achieved only if the infinite number of higher massive fields are present in the theory.

From the other hand, there are arguments that in string theory a general coordinate invariant effective field action reproducing the correct S-matrix both for massless and massive string states does not exist too [8]. The full effective action for all string fields is not general coordinate invariant and general covariance arises only as approximate symmetry in effective action for massless fields once all the massive fields are integrated out. That effective action for massive fields cannot be covariant follows, for examlple, from the fact that terms cubic in massive fields can contain only flat metric and there is no terms of higher powers [8].

Though influence of massive string modes is negligible at low energies it is desirable to understand how their consistent description in string theory arises. In addition to the general importance of this problem knowledge of effective equations of motion for massive string modes can be useful in various applications of string theory, for instance, in pre-Big Bang cosmology [9].

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[2] For description of higher spin massless fields on specific background see e.g. [6]

And indeed, as we show in this paper there exists a possibility to derive from the string theory covariant equations for massive higher spins fields interacting with background gravity linear in massive fields. This possibility does not contradict to non-covariance of the full string field action [8]. Terms in this action cubic in massive fields should really be non-covariant but one can derive in general covariant form terms quadratic in massive fields. The aim of our paper is to show explicitly how this procedure works using as an example dynamics of the second rank tensor from the first massive level of open bosonic string.

A convinient method of deriving effective field equations of motion from the string theory is provided by the $\sigma-$model approach [10]. Within this approch a string interacting with background fields is described by a two dimensional field theory and effective equations of motion arise from the requirement of quantum Weyl invariance. Perturbative derivation of these equations is well suited for massless string modes because the corresponding two dimensional theory is renormalizable and loop expansion corresponds to expansion of string effective action in powers of string length $\sqrt{\alpha'}$.

Inclusion of interaction with massive modes [11]–[13] makes the theory non-renormalizable but this fact does not represent a problem since in string theory one considers the whole infinite set of massive fields. Infinite number of counterterms needed for cancellation of divergences generating by a specific massive field in classical action leads to renormalization of an infinite number of massive fields. The only property of the theory crucial for possibility of derivation of perturbative information is that number of massive fields giving contributions to renormalization of the given field should be finite. As was shown in [12] string theory does fulfill this requirement. To calculate $\beta-$function for any massive field it is sufficient to find divergences coming only from a finite number of other massive fields and thus it is possible to derive perturbatively effective equations of motion for any background fields in any order in α'. To find non-perturbative contribution to the effective equations of motion one should use, for example, the method of exact renormalization group [14]. But this method has a serious disadvantage as it requires explicit separation of classical action into free and interaction parts and thus leads to non-covariant equations for background fields. From the general point of view this fact does not contradict general properties of string theory but makes it rather difficult to establish relations between string fields equations and ordinary field theory.

In this paper we discuss derivation of covariant equations of motion for massive string fields interacting with gravity by means of ordinary perturbative analysis of quantum Weyl invariance condition in the corresponding $\sigma-$model. Of course, perturbatively we can obtain equations only linear in massive fields. It was noted long ago [8] that one can make such a field redefinition in the string effective action that terms quadratic in massive fields (linear terms in equations of motion) acquire dependence on arbitrary higher powers of massless fields and so may be covariant. In this paper we receive explicitly these interaction terms in the lowest in α' approximation.

As a model for our calculations we use bosonic open string theory interacting with background fields of the massless and the first massive levels. First massive level contains symmetric second rank tensor which provides the simplest example of massive higher spin field interacting with gravity. We begin with analysing the problem of consistency for this field interacting with gravity from the point of view of field theory. Consistency of the theory requires that number of physical propagating degrees of freedom should be the same as in the flat space limit. We obtain the most general form of classical action fulfilling this requirement in arbitrary dimension and show that in case of actions quadratic in spacetime derivatives (i.e. linear in curvature) consistency can be achieved only on gravitational backgrounds with vanishing traceless part of Ricci tensor. If one includes in the action an infinite number of terms with all possible powers of curvature then consistency can be achieved without any restrictions on the background geometry and this is just the case realizied by string theory. In order to obtain equations of motion for the massive spin 2 field from the string theory we build effective action for the corresponding two dimensional theory, perform renormalization of background fields and composite operators and construct the renormalized operator of energy momentum tensor trace.

This rather standard scheme was first developed for calculations in closed string theory in massless background fileds [10, 15] and then was generalized for the open string theory [16] and for strings in massive fields [12].

Making perturbative calculations we restrict ourselves to string world sheets with topology of a disk. The resulting equations of motion for graviton will not contain dependence on massive fields from the open string spectrum because these fields interact only with the boundary of world sheet and can not influence the local physics in the bulk. For example, in the case of graviton and massive fields from the open string spectrum one expects that equations of motion for the graviton would look like ordinary vacuum Einstein equations without matter. One can obtain contributions from open string background fields to the right hand side of Einstein equations for gravity only considering world sheets of higher genus [17].

The organization of the paper is as follows. First we describe the most general consistent equations of motion for massive spin 2 field on curved manifolds from the point of view of ordinary field theory. Then we describe the string model that we use for derivations of effective equations of motion of string massive fields. Requirement of quantum Weyl invariance leads to effective string fields equations of motion and we show in the lowest order in α' that these equations are consistent.

It follows from the analysis of irreducible representaions of 4-dimensional Poincare group that massive spin 2 field in flat spacetime can be desribed by symmetric transverse and traceless tensor of the second rank $H_{\mu\nu}$ satisfying mass-shell condition:

$$\Big(\partial^2 - m^2\Big) H_{\mu\nu} = 0, \qquad \partial^\mu H_{\mu\nu} = 0, \qquad H^\mu{}_\mu = 0. \tag{1}$$

In higher dimensional spacetimes Poincare algebras have more than two Casimir operators and there are several different spins for $D > 4$. Talking about spin 2 massive field in arbitrary dimension we will mean, as usual, that this field by definition satisfies the same equations (1) as in $D = 4$. After dimensional reduction to $D = 4$ such a field will describe massive spin two representation of $D = 4$ Poincare algebra plus infinite tower of Kaluza-Klein descedants.

It is well known that all the equations (1) can be derived from the Fierz-Pauli action:

$$\begin{aligned} S &= \int d^D x \Big\{ \frac{1}{4}\partial_\mu H \partial^\mu H - \frac{1}{4}\partial_\mu H_{\nu\rho}\partial^\mu H^{\nu\rho} - \frac{1}{2}\partial^\mu H_{\mu\nu}\partial^\nu H + \frac{1}{2}\partial_\mu H_{\nu\rho}\partial^\rho H^{\nu\mu} \\ &\qquad - \frac{m^2}{4} H_{\mu\nu} H^{\mu\nu} + \frac{m^2}{4} H^2 \Big\}. \end{aligned} \tag{2}$$

where $H = \eta^{\mu\nu} H_{\mu\nu}$.

Equations of motion following from the action (2)

$$\begin{aligned} E_{\mu\nu} &= \partial^2 H_{\mu\nu} - \eta_{\mu\nu}\partial^2 H + \partial_\mu\partial_\nu H + \eta_{\mu\nu}\partial^\alpha\partial^\beta H_{\alpha\beta} - \partial_\sigma\partial_\mu H^\sigma{}_\nu - \partial_\sigma\partial_\nu H^\sigma{}_\mu \\ &\qquad - m^2 H_{\mu\nu} + m^2 H \eta_{\mu\nu} = 0 \end{aligned} \tag{3}$$

can be used to build $D+1$ expressions without second derivatives of $H_{\mu\nu}$:

$$\partial^\mu E_{\mu\nu} = m^2 \partial_\nu H - m^2 \partial^\mu H_{\mu\nu} = 0 \tag{4}$$

$$\frac{m^2}{D-2}\eta^{\mu\nu} E_{\mu\nu} + \partial^\mu\partial^\nu E_{\mu\nu} = H m^4 \frac{D-1}{D-2} = 0 \tag{5}$$

These expressions represent constraints on the initial values for the field $H_{\mu\nu}$ and its first derivatives. Thus the theory contains the same local dynamical degrees of freedom as the system (1) and describes traceless and transverse symmetric tensor field of the second rank.

Now if we want to construct a theory of massive spin 2 field on a curved manifold we should provide the same number of propagating degrees of freedom as in the flat case. It means that one should be able to build exactly $D+1$ constraints from the equations of motion $E_{\mu\nu}$ and in the flat spacetime limit these constraints should reduce to (4,5).

Generalizing (2) to curved spacetime we should substitute all derivatives for the covariant ones and also we can add non-minimal terms containing curvature tensor. As a result, the most general action for massive spin 2 field in curved spacetime quadratic in derivatives and consistent with the flat limit should have the form [1]

$$S = \int d^D x \sqrt{-G} \Big\{ \frac{1}{4}\nabla_\mu H \nabla^\mu H - \frac{1}{4}\nabla_\mu H_{\nu\rho}\nabla^\mu H^{\nu\rho} - \frac{1}{2}\nabla^\mu H_{\mu\nu}\nabla^\nu H + \frac{1}{2}\nabla_\mu H_{\nu\rho}\nabla^\rho H^{\nu\mu}$$

$$
\begin{aligned}
&+\frac{a_1}{2}RH_{\alpha\beta}H^{\alpha\beta}+\frac{a_2}{2}RH^2+\frac{a_3}{2}R^{\mu\alpha\nu\beta}H_{\mu\nu}H_{\alpha\beta}+\frac{a_4}{2}R^{\alpha\beta}H_{\alpha\sigma}H_\beta{}^\sigma+\frac{a_5}{2}R^{\alpha\beta}H_{\alpha\beta}H \\
&-\frac{m^2}{4}H_{\mu\nu}H^{\mu\nu}+\frac{m^2}{4}H^2\Big\}
\end{aligned}
\tag{6}
$$

where $a_1, \ldots a_5$ are so far arbitrary coefficients, $R^\mu{}_{\nu\lambda\kappa}=\partial_\lambda\Gamma^\mu_{\nu\kappa}-\ldots$; $R_{\mu\nu}=R^\lambda_{\mu\lambda\nu}$.

Equations of motion

$$
\begin{aligned}
E_{\mu\nu} \;=\;& \nabla^2 H_{\mu\nu}-G_{\mu\nu}\nabla^2 H+\nabla_\mu\nabla_\nu H+G_{\mu\nu}\nabla^\alpha\nabla^\beta H_{\alpha\beta}-\nabla_\sigma\nabla_\mu H^\sigma{}_\nu-\nabla_\sigma\nabla_\nu H^\sigma{}_\mu \\
&+2a_1RH_{\mu\nu}+2a_2G_{\mu\nu}RH+2a_3R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta}+a_4R_\mu{}^\alpha H_{\alpha\nu}+a_4R_\nu{}^\alpha H_{\alpha\mu} \\
&+a_5R_{\mu\nu}H+a_5G_{\mu\nu}R^{\alpha\beta}H_{\alpha\beta}-m^2H_{\mu\nu}+m^2HG_{\mu\nu}=0
\end{aligned}
\tag{7}
$$

should contain one vector constraint and one scalar constraint generalizing (4,5) for the case of curved background.

There are no problems with generalization of the vector constraint (4). It does not contain second derivatives of $H_{\mu\nu}$ for any gravitational background and for any values of coefficients $a_1, \ldots a_5$:

$$
\begin{aligned}
\nabla^\mu E_{\mu\nu} \;=\;& 2a_1R\nabla^\mu H_{\mu\nu}+2a_2R\nabla_\nu H+2a_3R^{\mu\alpha}{}_\nu{}^\beta\nabla_\mu H_{\alpha\beta}+a_4R^{\mu\alpha}\nabla_\mu H_{\alpha\nu} \\
&+(a_4-2)R^\alpha{}_\nu\nabla^\mu H_{\alpha\mu}+a_5R^{\alpha\mu}\nabla_\nu H_{\alpha\mu}+(a_5+1)R^\alpha{}_\nu\nabla_\alpha H \\
&-m^2\nabla^\mu H_{\mu\nu}+m^2\nabla_\nu H+\ldots=0
\end{aligned}
\tag{8}
$$

where dots stand for the terms not containing derivatives of the field $H_{\mu\nu}$.

As for the curved space generalization of the scalar constraint (5), it can have additional terms proportional to curvature and does contain second derivatives:

$$
\begin{aligned}
&\frac{m^2}{D-2}G^{\mu\nu}E_{\mu\nu}+\nabla^\mu\nabla^\nu E_{\mu\nu}+b_1RG^{\mu\nu}E_{\mu\nu}+b_2R^{\mu\nu}E_{\mu\nu}= \\
&=\Big(2a_1+b_1(D-2)+b_2-\frac{2a_3}{D(D-1)}+\frac{2a_4-2b_2-2}{D}\Big)R\nabla^\alpha\nabla^\beta H_{\alpha\beta} \\
&\quad+\Big(2a_2-b_1(D-2)-b_2+\frac{2a_3}{D(D-1)}+\frac{2a_5+2b_2+1}{D}\Big)R\nabla^2 H \\
&\quad+2a_3C^{\mu\alpha\nu\beta}\nabla_\mu\nabla_\nu H_{\alpha\beta}+\Big(2a_4-2b_2-2-\frac{4a_3}{D-2}\Big)\tilde{R}^{\alpha\beta}\nabla_\alpha\nabla^\mu H_{\mu\beta} \\
&\quad+\Big(a_5+b_2+\frac{2a_3}{D-2}\Big)\tilde{R}^{\alpha\beta}\nabla^2 H_{\alpha\beta}+\Big(a_5+b_2+1+\frac{2a_3}{D-2}\Big)\tilde{R}^{\alpha\beta}\nabla_\alpha\nabla_\beta H+\ldots=0
\end{aligned}
\tag{9}
$$

where dots stand for the terms without second derivatives of $H_{\mu\nu}$ and we decomposed the curvature tensor $R_{\mu\nu\alpha\beta}$ into Weyl tensor $C_{\mu\nu\alpha\beta}$, traceless part of the Ricci tensor $\tilde{R}_{\mu\nu}=R_{\mu\nu}-\frac{1}{D}G_{\mu\nu}R$ and scalar curvature. It is impossible to cancel all second derivatives in (9) by fixing the coefficients a_i since at least some combination of the last two terms with $\tilde{R}^{\alpha\beta}$ will always remain. So in order to achieve consistency with the flat space limit one has to impose restriction on the gravitational background

$$
R_{\mu\nu}=\frac{1}{D}G_{\mu\nu}R \tag{10}
$$

thus cancelling bad terms with second derivatives of $H_{\mu\nu}$ in (9). This restriction is no very strong and allows one to consider a wide class of curved backgrounds. Note, that in all previous works only the constant curvature D=4 spacetimes were considered. The condition (10) means that only traceless part of Ricci tensor should vanish while scalar curvature and Weyl tensor can be arbitrary. It is automatically fulfilled for $D=2$ where, of course, there are no problems of consistent interaction of higher spin fields at all.

In fact, this condition can be yet more weakened because in order to achieve consistency one should cancel in (9) only second *time* derivatives. For example, if we are dealing with a stationary spacetime possessing a timelike Killing vector k^μ, $k^2<0$ it is sufficient to demand:

$$
k^\mu k^\nu R_{\mu\nu}=\frac{1}{D}k^2R
$$

which means that only projection of the traceless part of $R_{\mu\nu}$ on orbits of k^μ should vanish.

Other terms with second derivatives in (9) can be cancelled by choosing appropriate coefficients. As a result, one arrives at one-parameter family of consistent theories for the massive spin-2 fields on the spacetime fulfilling (10):

$$S = \int d^D x \sqrt{-G} \Big\{ \frac{1}{4} \nabla_\mu H \nabla^\mu H - \frac{1}{4} \nabla_\mu H_{\nu\rho} \nabla^\mu H^{\nu\rho} - \frac{1}{2} \nabla^\mu H_{\mu\nu} \nabla^\nu H + \frac{1}{2} \nabla_\mu H_{\nu\rho} \nabla^\rho H^{\nu\mu} + \frac{\xi}{2D} R H_{\mu\nu} H^{\mu\nu} + \frac{1-2\xi}{4D} R H^2 - \frac{m^2}{4} H_{\mu\nu} H^{\mu\nu} + \frac{m^2}{4} H^2 \Big\}. \tag{11}$$

ξ is the only dimensionless coupling constant responsible for non-minimality of interaction with curved background. This is the most general form of the action for the massive spin 2 field in curved spacetime of arbitrary dimension leading to consistent equations of motion. Previous works [1]-[3] treated only cases with various particular values of ξ and only for $D = 4$.

Equations of motion and constraints have the following form:

$$\begin{aligned} E_{\mu\nu} &= \nabla^2 H_{\mu\nu} - G_{\mu\nu} \nabla^2 H + \nabla_\mu \nabla_\nu H + G_{\mu\nu} \nabla^\alpha \nabla^\beta H_{\alpha\beta} - \nabla_\sigma \nabla_\mu H^\sigma{}_\nu - \nabla_\sigma \nabla_\nu H^\sigma{}_\mu \\ &\quad + \frac{2\xi}{D} R H_{\mu\nu} + \frac{1-2\xi}{2D} R H G_{\mu\nu} - m^2 H_{\mu\nu} + m^2 H G_{\mu\nu} = 0 \end{aligned} \tag{12}$$

$$\begin{aligned} \nabla^\mu E_{\mu\nu} &= \frac{2(1-\xi)}{D} \Big(R \nabla_\nu H + H \nabla_\nu R + R \nabla^\mu H_{\mu\nu} + H_{\mu\nu} \nabla^\mu R \Big) \\ &\quad - m^2 \nabla^\mu H_{\mu\nu} + m^2 \nabla_\nu H = 0 \end{aligned} \tag{13}$$

$$\begin{aligned} & \frac{m^2}{D-2} G^{\mu\nu} E_{\mu\nu} + \nabla^\mu \nabla^\nu E_{\mu\nu} + \frac{2(1-\xi)}{D(D-2)} R G^{\mu\nu} E_{\mu\nu} = \\ &\quad = \frac{2(1-\xi)}{D} \Big(2 \nabla^\mu R \nabla_\mu H + H \nabla^2 R - 2 \nabla^\mu R \nabla^\nu H_{\mu\nu} - H_{\mu\nu} \nabla^\mu \nabla^\nu R \Big) \\ &\quad + H \Big(m^4 \frac{D-1}{D-2} + m^2 R \frac{3D - 2 + 2\xi(1-D)}{D(D-2)} + R^2 \frac{2(1-\xi)(D + 2\xi(1-D))}{D^2(D-2)} \Big) = 0 \end{aligned} \tag{14}$$

Note that consistency of the theory (in the sense that it has correct flat limit) cannot be achieved with minimal coupling to gravity. For any value of ξ there are non-minimal terms in (11).

The constraints (13,14) have the simplest form when $\xi = 1$. The equations of motion in this case reduce to:

$$\begin{aligned} & \nabla^2 H_{\mu\nu} + 2 R^\alpha{}_\mu{}^\beta{}_\nu H_{\alpha\beta} - m^2 H_{\mu\nu} = 0, \\ & H^\mu{}_\mu = 0, \qquad \nabla^\mu H_{\mu\nu} = 0, \end{aligned} \tag{15}$$

The theory is consistent for any other values of ξ, only explicit form of constraints and dynamical equations of motion is more complicated than that of (15).

Of course, one would like to overcome the restriction (10) on the background gravitational field and to construct consistent theory of the massive spin 2 in arbitrary curved spacetime. Such a possibility really exists if one considers a lagrangian which contains an infinite number of terms. It is possible only for massive fields because the mass of the field $H_{\mu\nu}$ is the only dimensionful parameter which can be used to construct a lagrangian with terms of arbitrary orders in curvature multiplied by the corresponding powers of $1/m^2$:

$$\begin{aligned} S_H = \int d^D x \sqrt{-G} \Big\{ & \frac{1}{4} \nabla_\mu H \nabla^\mu H - \frac{1}{4} \nabla_\mu H_{\nu\rho} \nabla^\mu H^{\nu\rho} - \frac{1}{2} \nabla^\mu H_{\mu\nu} \nabla^\nu H + \frac{1}{2} \nabla_\mu H_{\nu\rho} \nabla^\rho H^{\nu\mu} \\ & + \frac{a_1}{2} R H_{\alpha\beta} H^{\alpha\beta} + \frac{a_2}{2} R H^2 + \frac{a_3}{2} R^{\mu\alpha\nu\beta} H_{\mu\nu} H_{\alpha\beta} + \frac{a_4}{2} R^{\alpha\beta} H_{\alpha\sigma} H_\beta{}^\sigma + \frac{a_5}{2} R^{\alpha\beta} H_{\alpha\beta} H \\ & + \frac{1}{m^2} (R \nabla H \nabla H + R H \nabla \nabla H + R R H H) + O\Big(\frac{1}{m^4}\Big) - \frac{m^2}{4} H_{\mu\nu} H^{\mu\nu} + \frac{m^2}{4} H^2 \Big\} \end{aligned} \tag{16}$$

Actions of this kind are expected to arise naturally in string theory where the role of mass parameter is played by string tension $m^2 = 1/\alpha'$ and perturbation theory in α' gives effective actions of the form (16)[3].

Possibility of constructing consistent equations for massive higher spin fields as series in curvature was recently studied in [5] where for spin 2 field these equations were derived in particular case of symmetrical Einstein spaces in linear in curvature order. In principle, as we argue below consistency can be achieved without imposing any restrictions on gravitational background, at least in the lowest order in $1/m^2$.

Equations of motion are:

$$\begin{aligned} E_{\mu\nu} \;\; &= \;\; \nabla^2 H_{\mu\nu} - G_{\mu\nu}\nabla^2 H + \nabla_\mu\nabla_\nu H + G_{\mu\nu}\nabla^\alpha\nabla^\beta H_{\alpha\beta} - \nabla_\sigma\nabla_\mu H^\sigma{}_\nu - \nabla_\sigma\nabla_\nu H^\sigma{}_\mu \\ &+ 2a_1 R H_{\mu\nu} + 2a_2 G_{\mu\nu} R H + 2a_3 R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta} + a_4 R_\mu{}^\alpha H_{\alpha\nu} + a_4 R_\nu{}^\alpha H_{\alpha\mu} \\ &+ a_5 R_{\mu\nu} H + a_5 G_{\mu\nu} R^{\alpha\beta} H_{\alpha\beta} + \frac{1}{m^2}(R\nabla\nabla H + \nabla R\nabla H + \nabla\nabla R H + RRH) \\ &+ O\left(\frac{1}{m^4}\right) - m^2 H_{\mu\nu} + m^2 H G_{\mu\nu} = 0 \end{aligned} \tag{17}$$

Now we solve perturbatively the constraints with respect to the trace and longitudinal part of $H_{\mu\nu}$. Of course, the combinations of equations of moition $E_{\mu\nu}$ forming the constraints can change in higher orders but this is irrelevant for the lowest order conditions. The vector constraint arise from:

$$\begin{aligned} &\nabla^\mu E_{\mu\nu} + \frac{1}{m^2}(R\nabla E + \nabla R E) + O\left(\frac{1}{m^4}\right) = -m^2\nabla^\mu H_{\mu\nu} + m^2\nabla_\nu H + O(1) = 0 \qquad \Rightarrow \\ &\qquad\Rightarrow \qquad \nabla^\mu H_{\mu\nu} - \nabla_\nu H = O\left(\frac{1}{m^2}\right) \end{aligned} \tag{18}$$

Since the trace enters the equations of motion in the term $m^2 H$ we will need expansion of the scalar constraint up to the next to leading terms:

$$\begin{aligned} &\frac{m^2}{D-2} G^{\mu\nu} E_{\mu\nu} + \nabla^\mu\nabla^\nu E_{\mu\nu} + b_1 R G^{\mu\nu} E_{\mu\nu} + b_2 R^{\mu\nu} E_{\mu\nu} + O\left(\frac{1}{m^2}\right) = \\ &= m^4 \frac{D-1}{D-2} H + m^2 R H \Big[\frac{2a_1 + 2Da_2 + a_5}{D-2} + (D-1)b_1 + b_2\Big] \\ &\quad + m^2 R^{\alpha\beta} H_{\alpha\beta} \Big[\frac{2a_3 + 2a_4 + Da_5}{D-2} - b_2\Big] + O(1) = 0, \end{aligned} \tag{19}$$

We see that in the lowest orders the expression (19) does not contain second derivatives for any values of the coefficients a_i, b_i and for arbitrary background. Solving it with respect to the trace we get

$$H = \frac{1}{m^2} R^{\alpha\beta} H_{\alpha\beta} \frac{(D-2)b_2 - 2a_3 - 2a_4 - Da_5}{D-1} + O\left(\frac{1}{m^4}\right) \tag{20}$$

Substituting (20) back to the equations of motion we get in the lowest order the system of equations for $H_{\mu\nu}$ equivalent to the original equations (17):

$$\begin{aligned} &\nabla^2 H_{\mu\nu} + 2a_1 R H_{\mu\nu} + (2a_3 + 2) R^\alpha{}_\mu{}^\beta{}_\nu H_{\alpha\beta} + 2(a_4 - 1) R^\sigma{}_{(\mu} H_{\nu)\sigma} \\ &\quad + G_{\mu\nu} R^{\alpha\beta} H_{\alpha\beta} \frac{(D-2)b_2 - 2a_3 - 2a_4 - a_5}{D-1} - m^2 H_{\mu\nu} + O\left(\frac{1}{m^2}\right) = 0, \\ &\nabla^\mu H_{\mu\nu} + O\left(\frac{1}{m^2}\right) = 0, \qquad H_\mu{}^\mu + O\left(\frac{1}{m^2}\right) = 0. \end{aligned} \tag{21}$$

Mass shell condition in (21) contains in this order four non-minimal terms with curvature tensor and coefficients at all of them are arbitrary.

[3] In fact, string theory should lead to even more general effective actions than (16) since in the higher α' corrections higher derivatives of the field $H_{\mu\nu}$ should appear.

This arbitrariness is related to the possibility of making field redefinitions in the action (16). It is well known [18] that in string theory such possibility allows to change effective action for massless background fields in higher orders. In case of massive fields action contains the mass term which is of order $1/\alpha'$ and this leads to possibility of changing some coefficient in the effective action already in the lowest order. Namely, since (16) contains all higher powers of $1/m^2$ one can redefine the field $H_{\mu\nu}$ through an infinite series as:

$$H_{\mu\nu} \to H_{\mu\nu} + \frac{1}{m^2}\left(\lambda_1 R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta} + \lambda_2 R^\alpha{}_{(\mu} H_{\nu)\alpha} + \lambda_3 R H_{\mu\nu} + \lambda_4 G_{\mu\nu} R^{\alpha\beta} H_{\alpha\beta}\right) + O\left(\frac{1}{m^4}\right) \tag{22}$$

Using such redefinitions one can assign any values to the coefficients a_1, a_3, a_4, b_2 in (21) and so specific values of these coefficients do not play any role in achieving consistency of the theory (16). Requirement of consistency will restrict parameters of the theory only in higher orders in $1/m^2$.

We would like to stress once more that the theory (16) admits any gravitational background and so no inconsistencies arise if one treats gravity as dynamical field satisfying Einstein equations with the energy - momentum tensor for the field $H_{\mu\nu}$. This system is consistent in the sense that it has correct number of degrees of freedom. One can also study additional requirements the theory should fulfill, e.g. causality [1] or tree level unitarity of graviton - massive spin 2 field interaction [3]. These requirements can lead to some additional restrictions on the parameters of the theory.

Now we consider sigma-model description of an open string interacting with two background fields – massless graviton $G_{\mu\nu}$ and second rank symmetric tensor field $H_{\mu\nu}$ from the first massive level of the open string spectrum. We are going to show that effective equations of motion for these fields are of the form (21) and explicitly calculate all the coefficients in these equations in the lowest order in α'.

Classical action has the form

$$S = S_0 + S_I = \frac{1}{4\pi\alpha'}\int_M d^2z\sqrt{g}g^{ab}\partial_a x^\mu \partial_b x^\nu G_{\mu\nu} + \frac{1}{2\pi\alpha'\mu}\int_{\partial M} dt\; eH_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \tag{23}$$

Here $\mu,\nu = 0,\ldots,D-1$; $a,b = 0,1$ and we introduced the notation $\dot{x}^\mu = \frac{dx^\mu}{edt}$. The first term S_0 is an integral over two-dimensional string world sheet M with metric g_{ab} and the second S_I represents a one-dimensional integral over its boundary with einbein e. We work in euclidian signature and restrict ourselves to flat world sheets with straight boudaries. It means that both two-dimensional scalar curvature and extrinsic curvature of the world sheet boundary vanish and we can always choose such coordinates that $g_{ab} = \delta_{ab}$, $e = 1$.

Theory has two dimensionful parameters. α' is fundamental string length squared, D-dimensional coordinates x^μ have dimension $\sqrt{\alpha'}$. Another parameter μ carries dimension of inverse length in two-dimensional field theory (23) and plays the role of renormalization scale. It is introduced in (23) to make the background field $H_{\mu\nu}$ dimensionless. In fact, power of μ is responsible for the number of massive level to which a background field belongs because one expects that open string interacts with a field from n-th massive level by means of the term

$$\mu^{-n}(\alpha')^{-\frac{n+1}{2}}\int_{\partial M} dt\; e\dot{x}^{\mu_1}\ldots\dot{x}^{\mu_{n+1}}H_{\mu_1\ldots\mu_{n+1}}(x)$$

The action (23) is non-renormalizable from the point of view of two-dimensional quantum field theory. Inclusion of interaction with any massive background produces in each loop an infinite number of divergencies and requires an infinite number of different massive fields in the action. But massive modes from the $n-$th massive level give vertices proportional to μ^{-n} and so they cannot contribute to renormalization of fields from lower levels. Of course, this argument supposes that we treat the theory perturbatively defining propagator for X^μ only by the graviton term in (23).

Varying (23) one gets classical equations of motion with boundary conditions:

$$\begin{aligned}
&g^{ab}D_a\partial_b x^\alpha \equiv g^{ab}(\partial_a\partial_b x^\alpha + \Gamma^\alpha_{\mu\nu}(G)\partial_a x^\mu \partial_b x^\nu) = 0,\\
&G_{\mu\nu}\partial_n x^\mu|_{\partial M} - \frac{2}{\mu}\mathcal{D}_t^2 x^\mu H_{\mu\nu} + \frac{1}{\mu}\dot{x}^\mu\dot{x}^\lambda(\nabla_\nu H_{\mu\lambda} - \nabla_\mu H_{\nu\lambda} - \nabla_\lambda H_{\mu\nu}) = 0
\end{aligned} \tag{24}$$

where $\partial_n = n^a\partial_a$, n^a – unit inward normal vector to the world sheet boundary and $\mathcal{D}_t^2 x^\mu = \ddot{x}^\mu + \Gamma^\mu_{\nu\lambda}(G)\dot{x}^\nu\dot{x}^\lambda$.

Divergent part of the one loop effective action has the form

$$\Gamma^{(1)}_{div} = \frac{\mu^{-\varepsilon}}{4\pi\varepsilon}\int_M d^{2+\varepsilon}z\sqrt{g}g^{ab}\partial_a x^\mu \partial_b x^\nu R_{\mu\nu}$$
$$- \frac{\mu^{-\varepsilon-1}}{2\pi\varepsilon}\int_{\partial M} dte(t)\dot{x}^\mu\dot{x}^\nu \left(\nabla^2 H_{\mu\nu} - 2R_\mu{}^\alpha H_{\alpha\nu} + R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta}\right) + O(\mu^{-2}) \tag{25}$$

where the terms $O(\mu^{-2})$ give contributions to renormalization of only the second and higher massive levels. Hence one-loop renormalization of the background fields looks like:

$$\begin{aligned}\overset{\circ}{G}_{\mu\nu} &= \mu^\varepsilon G_{\mu\nu} - \frac{\alpha'\mu^\varepsilon}{\varepsilon}R_{\mu\nu}\\ \overset{\circ}{H}_{\mu\nu} &= \mu^\varepsilon H_{\mu\nu} + \frac{\alpha'\mu^\varepsilon}{\varepsilon}\left(\nabla^2 H_{\mu\nu} - 2R^\sigma{}_{(\mu}H_{\nu)\sigma} + R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta}\right)\end{aligned} \tag{26}$$

with circles denoting bare values of the fields. We would like to stress once more that higher massive levels do not influence the renormalization of any given field from the lower massive levels and so the result (26) represents the full answer for perturbative one-loop renormalization of $G_{\mu\nu}$ and $H_{\mu\nu}$.

Now to impose the condition of Weyl invariance of the theory at the quantum level we calculate the trace of energy momentum tensor in $d = 2+\varepsilon$ dimension:

$$T(z) = g_{ab}(z)\frac{\delta S}{\delta g_{ab}(z)} = \frac{\varepsilon\mu^{-\varepsilon}}{8\pi\alpha'}g^{ab}(z)\partial_a x^\mu\partial_b x^\nu G_{\mu\nu} - \frac{\mu^{-1-\varepsilon}}{4\pi\alpha'}H_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\delta_{\partial M}(z) \tag{27}$$

and perform one-loop renormalization of the composite operators:

$$\left(\dot{x}^\mu\dot{x}^\nu\ \overset{\circ}{H}_{\mu\nu}\right)_0 = \mu^{-\varepsilon}\left[\dot{x}^\mu\dot{x}^\nu H_{\mu\nu}\right] \tag{28}$$

$$\begin{aligned}(g^{ab}\partial_a x^\mu\partial_b x^\nu\ \overset{\circ}{G}_{\mu\nu})_0 &= \mu^\varepsilon\Big[g^{ab}\partial_a x^\mu\partial_b x^\nu(G_{\mu\nu} - \frac{\alpha'}{\varepsilon}R_{\mu\nu})\Big]\\ &+ \frac{\alpha'\mu^{-1+\varepsilon}}{\varepsilon}\left[H_\alpha{}^\alpha\delta''_{\partial M}(z) + \mathcal{D}_t^2 x^\mu(\nabla_\mu H_\alpha{}^\alpha - 4\nabla^\alpha H_{\alpha\mu})\delta_{\partial M}(z)\right.\\ &\left.+\dot{x}^\mu\dot{x}^\nu(\nabla_\mu\nabla_\nu H_\alpha{}^\alpha - 4\nabla^\alpha\nabla_{(\mu}H_{\nu)\alpha} + 2\nabla^2 H_{\mu\nu} - 2R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta})\delta_{\partial M}(z)\right]\end{aligned} \tag{29}$$

Here delta-function of the boundary $\delta_{\partial M}(z)$ is defined as

$$\int_M \delta_{\partial M}(z)V(z)\sqrt{g(z)}d^2z = \int_{\partial M} V|_{z\in\partial M}e(t)dt \tag{30}$$

The renormalized operator of the energy momentum tensor trace is:

$$\begin{aligned}8\pi[T] &= -\Big[g^{ab}\partial_a x^\mu\partial_b x^\nu E^{(0)}_{\mu\nu}(x)\Big] + \frac{2}{\mu}\delta_{\partial M}(z)\Big[\dot{x}^\mu\dot{x}^\nu E^{(1)}_{\mu\nu}(x)\Big]\\ &+ \frac{1}{\mu}\delta_{\partial M}(z)\Big[\mathcal{D}_t^2 x^\mu E^{(2)}_\mu(x)\Big] + \frac{1}{\mu}\delta''_{\partial M}(z)\Big[E^{(3)}(x)\Big]\end{aligned} \tag{31}$$

where

$$\begin{aligned}E^{(0)}_{\mu\nu}(x) &= R_{\mu\nu} + O(\alpha')\\ E^{(1)}_{\mu\nu}(x) &= \nabla^2 H_{\mu\nu} - \nabla^\alpha\nabla_\mu H_{\alpha\nu} - \nabla^\alpha\nabla_\nu H_{\alpha\mu} - R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta} + \frac{1}{2}\nabla_\mu\nabla_\nu H_\alpha{}^\alpha - \frac{1}{\alpha'}H_{\mu\nu} + O(\alpha')\\ E^{(2)}_\mu(x) &= \nabla_\mu H_\alpha{}^\alpha - 4\nabla^\alpha H_{\alpha\mu} + O(\alpha')\\ E^{(3)}(x) &= H_\alpha{}^\alpha + O(\alpha')\end{aligned} \tag{32}$$

Terms of order $O(\alpha')$ arise from the higher loops contributions.

The requirement of quantum Weyl invariance tells that all $E(x)$ should vanish and so they are interpreted as effective equations of motion for background fields. They contain vacuum Einstein equation for graviton (in the lowest order in α'), curved spacetime generalization of the mass shell condition for the field $H_{\mu\nu}$ with the mass $m^2 = (\alpha')^{-1}$ and $D+1$ additional constraints on the values of this fields and its first derivatives. In fact, Einstein equations should not be vacuum ones but contain dependence on the field $H_{\mu\nu}$ through its enegry - momentum tensor $T^H_{\mu\nu}$. Our calculations could not produce this dependence because it is expected to arise only if one takes into account string world sheets with non-trivial topology and renormalizes new divergencies arising from string loops contribution [17]. Considering this fact we can write our final equations arising from the Weyl invariance of string theory in the form:

$$\begin{aligned}
&\nabla^2 H_{\mu\nu} + R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta} - R^\alpha{}_\mu H_{\alpha\nu} - R^\alpha{}_\nu H_{\alpha\mu} - \frac{1}{\alpha'} H_{\mu\nu} + O(\alpha') = 0, \\
&\nabla^\alpha H_{\alpha\nu} + O(\alpha') = 0, \qquad H^\mu{}_\mu + O(\alpha') = 0, \\
&R_{\mu\nu} + O(\alpha') = T^H_{\mu\nu} - \frac{1}{D-2} T^{H\alpha}{}_\alpha
\end{aligned} \tag{33}$$

This is a system of consistent equations of motion derived in the lowest order but if one wants to determine whether they can be deduced from an effective lagrangian (and to find this lagrangian) then one-loop contributions present in (33) are not sufficient. Comparing the effective equations of motion (32,33) with the general form of equations for spin 2 field discussed above we see that coefficients $E^{(i)}$ arising in the condition of string Weyl invariance are not equations directly following from a lagrangian (17) but some combinations of them analogous to (21). In order to reverse the procedure of passing from (17) to (21) one would need next to leading contributions in the conditions for $\nabla^\mu H_{\mu\nu}$ and $H^\mu{}_\mu$ (33).

Anyway the specific values of coefficients a_i in the effective action (16) are not important due to possibility of field redefinition (22). In string theory this possibility can be also interpreted as finite renormalization of the field $H_{\mu\nu}$. Our result (33) has coefficients specific for the minimal subtraction scheme which we used performing the renormalization of background fields and composite operators. Specific values of these coefficients are scheme dependent. In case of arbitrary renormalization scheme the first equation in (33) will take the form:

$$\begin{aligned}
\nabla^2 H_{\mu\nu} &+ c_1 R_\mu{}^\alpha{}_\nu{}^\beta H_{\alpha\beta} + c_2 R^\alpha{}_\mu H_{\alpha\nu} + c_2 R^\alpha{}_\nu H_{\alpha\mu} + c_3 R H_{\mu\nu} + c_4 G_{\mu\nu} R^{\alpha\beta} H_{\alpha\beta} \\
&- \frac{1}{\alpha'} H_{\mu\nu} + O(\alpha') = 0.
\end{aligned} \tag{34}$$

c_i are coefficients depending on the renormalization scheme and in this order in α' all of them can be made arbitrary by the field redefinition.

Let us summarize the obtained results. We investigated the problem of consistency of the equations of motion for spin-2 massive field in curved spacetime and found that two different description of this field are possible. First, for specific gravitational background satisfying (10) one can build an action leading to consistent equations including the tracelessness and transversality conditions. Another possibility (naturally arising in string theory) consists in building the theory as perturbation series in inverse mass. In the lowest order no consistency problems arise and equations of motion have the form (21).

Then we calculated the equations for the massive spin-2 background field arising in sigma model approach to string theory from the condition of quantum Weyl invariance in the lowest order in α'. The explicit form of the derived equations (33) appears to be a particular case of the general equations in field theory (21). General agreement between string theory and field theory is achieved if one takes into account a possibility of making field redefinitions in the effective action which in string theory calculations corresponds to possibility of making finite renormalization of background fields. We expect that in general in each order in α' the situation remains the same and it is possible to construct the part of string effective action quadratic in massive background field which should lead to generalized mass-shell and transversality conditions.

To determine this part of the bosonic string effective action completely one should also consider other massless background fields including the dilaton. This will require investigation of strings with curved world sheets and with non-vanishing extrinsic curvature on the boundary which complicates the sigma-model calculations. From the field theoretical point of view this will require to generalize the analysis of consistency for the massive spin 2 field propagating on background of both gravity and massless scalar field. We leave it for a future publication.

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